In the last two posts, we discussed how to easily handle constraints in linear arrangements. Today we will discuss how to handle constraints in circular arrangements, which are actually even simpler to sort out. Let's look at some examples.

Question 1: Seven people are to be seated at a round table. Andy and Bob don't want to sit next to each other. How many seating arrangements are possible?

Solution: There are 7 people who need to be seated around a circular table. Number of arrangements in which 7 people can be seated around a circular table = (7-1)! = 6!

(If you are not sure how we got this, check out this post.)

We need to find the number of arrangements in which two of them do not sit together. Let us instead find the number of arrangements in which they will sit together. We will then subtract these arrangements from the total 6! arrangements. Consider Andy and Bob to be one unit. Now we need to arrange 6 units around a round table. We can do this in 5! ways. But Andy and Bob can swap places so we need to multiply 5! by 2.

Number of arrangements in which Andy and Bob do sit next to each other = 2*5!
So, number of arrangements in which Andy and Bob don't sit next to each other = 6! – 2*5!

This is very similar to the way we handled such constraints in linear arrangements.

Question 2: There are 6 people, A, B, C, D, E and F. They have to sit around a circular table such that A cannot sit next to D and F at the same time. How many such arrangements are possible?

Solution: Total number of ways of arranging 6 people in a circle = 5! = 120

Now, A cannot sit next to D and F simultaneously.

Let's first find the number of arrangements in which A sits between D and F. In how many of these 120 ways will A be between D and F? Let's consider that D, A and F form a single unit. We make DAF sit on any three consecutive seats in 1 way and make other 3 people sit in 3! ways (since the rest of the 3 seats are distinct). But D and F can swap places so the number of arrangements will actually be 2*3! = 12

In all, we can make A sit next to D and F simultaneously in 12 ways.

The number of arrangements in which A is not next to D and F simultaneously is 120 – 12 = 108.

A slight variation of this question that would change the answer markedly is the following:

Question 3: There are 6 people, A, B, C, D, E and F. They have to sit around a circular table such that A can sit neither next to D nor next to F. How many such arrangements are possible?

Solution: In the previous question, A could sit next to D and F; the only problem was that A could not sit next to both of them at the same time. Here, A can sit next to neither D nor F. Generally, it is difficult to wrap your head around what someone cannot do. It is easier to consider what someone can do and go from there. A cannot sit next to D and F so he will sit next to two of B, C and E.

Let's choose two out of B, C and E. In other words, let's drop one of B, C and E. We can drop one of B, C and E in 3 ways (we can drop B or C or E). This means, we can choose two out of B, C and E in 3 ways (We will come back to choosing 2 people out of 3 when we work on combinations). Now, we can arrange the two selected people around A in 2 ways (say we choose B and C. We could have BAC or CAB). We make these three sit on any three consecutive seats in

1 way.

Number of ways of choosing two of B, C and E and arranging the chosen two with A = 3*2 = 6

The rest of the three people can sit in three distinct seats in 3! = 6 ways

Total number of ways in which A will sit next to only B, C or E (which means A will sit neither next to D nor next to F) = 6*6 = 36 ways

Now we will look at one last example.

Question 4: Six people are to be seated at a round table with seats arranged at equal distances. Andy and Bob don't want to sit directly opposite to each other. How many seating arrangements are possible?

Solution: Directly opposite means that Andy and Bob cannot sit at the endpoints of the diameter of the circular table.

Total number of arrangements around the circular table will be (6-1)! = 5!

But some of these are not acceptable since Andy sits opposite Bob in these. Let us see in how many cases Andy doesn't sit opposite Bob. Let's say we make Andy sit first. He can sit at the table in 1 way since all the seats are exactly identical for him. Now there are 5 seats left but Bob can take a seat in only 4 ways since he cannot occupy the seat directly opposite Andy. Now there are 4 people left and 4 distinct seats left so they can be occupied in 4! ways.

Total number of ways of arranging the 6 people such that Andy does not sit next to Bob = 1*4*4! = 96 arrangements.

Make sure you understand the logic used in this question. We will build up on it in the next post.